1. Consider an infinite solid, which in equilibrium can be thought of as a 3-D cubic lattice of identical particles of mass \( m \). A particle at \((x,y,z)\) is attached to its 6 neighbors at \((x+a,y,z)\), \((x-a,y,z)\), \((x,y+a,z)\), \((x,y-a,z)\), \((x,y,z+a)\), and \((x,y,z-a)\) by identical springs with spring constant \( k \), where \( a \) is the interparticle spacing on the lattice.

(a) Write down the discrete Lagrangian for this system, assuming the particle at the \((i,j,k)\) lattice site has displacement from equilibrium given by \((\eta_{ijk}, \xi_{ijk}, \zeta_{ijk})\).

(b) Take the limit \( a \to 0 \) and determine the Lagrangian density. Eliminate \( k \) by relating it to the bulk modulus \( B \) of the solid, defined by \( \Delta P = B(\Delta V/V) \), where \( \Delta P \) is the change in pressure due to a fractional change in volume. [Hint: \( B \) has dimensions of pressure.] Eliminate \( m \) by relating it to \( \rho \), the density of the solid.

(c) Apply the continuous form of Lagrange’s equations of motion to find the equations of motion for \( \eta \), \( \xi \), and \( \zeta \). Use your result to obtain the wave velocity in the solid.

2. (a) Show that if \( \psi \) and \( \psi^* \) are considered to be independent field variables, the Lagrangian density

\[
L = \frac{\hbar^2}{2m} \nabla \psi \cdot \nabla \psi^* + V \psi^* \psi + \frac{\hbar}{2i} (\psi^* \dot{\psi} - \psi \dot{\psi}^*)
\]

leads to the Schrödinger equation

\[
-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t},
\]

and also its complex conjugate.

(b) Find the elements of the stress–energy tensor. What are the canonical momenta?