1. Consider the hypothetical generating function \( F = F_5(Q_i, P_i, t) + \lambda \sum q_i P_i \) where \( \lambda \) is a constant. Under what conditions is this a proper generating function? Is it one of the four types of generating functions discussed in the text, or fundamentally different? When \( F \) is a proper generating function, describe its effect.

2. The Hamiltonian for a one-dimensional particle of mass \( m \) may be written as
\[
H = \frac{p^2}{2m} + V(x),
\]
where \( x \) is the usual coordinate and \( p \) the corresponding conjugate momentum.

(a) Find the generating function for the Galilean transformation \((x, p) \rightarrow (X, P)\) defined by \( X = x - v_0 t, P = p - mv_0 \).

(b) Find the new Hamiltonian of the system in terms of \( X \) and \( P \).

3. (a) Show that if \( a \) is a constant, then the transformation
\[
X = p + iax, \quad P = \frac{p - iax}{2ia},
\]
is canonical. Use the symplectic technique, i.e., show that \( MJM^T = J \), where \( M \) is the Jacobian matrix of the transformation.

(b) For the linear harmonic oscillator
\[
H = \frac{p^2}{2m} + \frac{kx^2}{2},
\]
choose \( a \) so that only one term remains in the new Hamiltonian (written in terms of \( X \) and \( P \)). Use Hamilton’s equations to solve for \( X \) and \( P \) as functions of time. Given initial conditions \( x = x_0 \) and \( p = p_0 \) at \( t = 0 \), find \( x \) and \( p \) as functions of time.

4. Evaluate the Poisson bracket \([Q, P]\) and find the conditions under which the following transformations are canonical.

(a) \( Q = q^\alpha \cos \beta p, \quad P = q^\alpha \sin \beta p \), where \( \alpha \) and \( \beta \) are constants.

(b) \( Q = \alpha p^{n_1} q^{n_2}, \quad P = \beta p^{n_3} q^{n_4} \), where \( \alpha, \beta, n_1, n_2, n_3, \) and \( n_4 \) are constants. Show that there is a three-dimensional family of transformations of this type, i.e., there is a family of transformations parametrized by three independent variables. Determine the range of allowed values for those variables.