1. Consider the following Lagrangian density

\[ \mathcal{L} = i\bar{\psi}\gamma_\mu \partial^\mu \psi - m\bar{\psi}\psi \]

where \( \psi \) is a four-component column vector.

(a) Show that this Lagrangian density is invariant under the infinitesimal gauge transformation \( \psi \rightarrow \psi e^{ie_\gamma} \).

(b) Find the conserved current associated with this symmetry.

2. Consider the following Lagrangian density

\[
\mathcal{L} = (\partial^\mu \phi_1^*) (\partial_\mu \phi_1) - \mu_1^2 \phi_1^* \phi_1 + (\partial^\mu \phi_2^*) (\partial_\mu \phi_2) - \mu_2^2 \phi_2^* \phi_2 + (\partial^\mu \phi_3^*) (\partial_\mu \phi_3) - \mu_3^2 \phi_3^* \phi_3 + g (\phi_1 \phi_2 \phi_3^* + \phi_1^* \phi_2^* \phi_3)
\]

where \( \phi_1, \phi_2, \) and \( \phi_3 \) are complex fields.

(a) Show that this Lagrangian density is invariant under the infinitesimal gauge transformation defined by \( \phi_1 \rightarrow \phi_1 e^{ie_\gamma}, \phi_2 \rightarrow \phi_2 e^{ie_\gamma}, \) and \( \phi_3 \rightarrow \phi_3 e^{-2ie_\gamma} \).

(b) Find the conserved current associated with this symmetry.