1. The coefficient of viscosity is \( \eta = (F/A)/(\Delta v/\Delta x) \), so

\[
m \frac{dv}{dt} = F = -\frac{\Delta v}{\Delta x} A \eta = -\frac{v}{\Delta x} A \eta
\]

where the minus sign indicates that the force on the board opposes the motion. Solving for \( v \) (with \( v(0) = v_0 \)):

\[
\ln\left(\frac{v}{v_0}\right) = -\frac{A \eta}{m \Delta x} t,
\]

\[
v = v_0 e^{-A \eta t/(m \Delta x)}.
\]

Solving for \( x \) (with \( x(0) = 0 \)):

\[
\frac{dx}{dt} = v = v_0 e^{-A \eta t/(m \Delta x)},
\]

\[
\int_0^x dx = \int_0^t v_0 e^{-A \eta t/(m \Delta x)} dt
\]

\[
x = -\frac{mv_0 \Delta x}{A \eta} e^{-A \eta t/(m \Delta x)} \bigg|_0^t
\]

\[
x = \frac{mv_0 \Delta x}{A \eta} \left(1 - e^{-A \eta t/(m \Delta x)}\right).
\]

The maximum value for \( x \) as \( t \to \infty \) is \( mv_0 \Delta x/(A \eta) \).

2. (a) The moment of inertia of the system is the sum of moments of inertia of the wheel plus mass:

\[
I = Mk^2 + m(2k)^2 = (M + 4m)k^2.
\]

(b) The potential energy is

\[
V(\theta) = -\int_0^\theta \tau \, d\theta
\]

where \( \tau \) is the net torque and the reference point is taken as \( \theta = 0 \) (this is analogous to \( V(x) = -\int F(x) \, dx \) for linear forces). The torque due to the spring is \( \tau = -K \theta \) and the torque due to gravity is \( mgr \sin \theta = mg(2k) \sin \theta \). Note that the spring wants to restore \( \theta \) back to zero, while gravity wants to move \( \theta \) away from zero (since once the mass is moved away from \( \theta = 0 \), gravity tends to pull it further away from equilibrium). So the net torque is \( \tau = -K \theta + 2mgk \sin \theta \) and

\[
V(\theta) = \int_0^\theta (K \theta - 2mgk \sin \theta) \, d\theta
\]

\[
= 2mgk \cos \theta + \frac{1}{2} K \theta^2 - 2mgk.
\]
The constant term is irrelevant, and would be different if another reference point had been chosen. For \( K = 4mgk/\pi \) we have

\[
V(\theta) = 2mgk \left( \cos \theta + \frac{\theta^2}{\pi} - 1 \right).
\]

To find equilibrium points, we set \( dV/d\theta = 0 \):

\[
0 = dV/d\theta = 2mgk \left( -\sin \theta + \frac{2}{\pi} \theta \right)
\]

\[
\theta = \frac{\pi}{2} \sin \theta.
\]

The last line has solutions \( \theta = -\pi/2, 0, \pi/2 \), so these are the equilibrium points. See the diagram for a sketch of the potential.

To see if the equilibrium points are stable, we must examine \( d^2V/d\theta^2 \):

\[
d^2V/d\theta^2 = 2mgk \left( -\cos \theta + \frac{2}{\pi} \right).
\]

At \( \theta = 0 \), \( d^2V/d\theta^2 < 0 \) (since \( 2/\pi < 1 \)) and this equilibrium point is unstable. At \( \theta = \pm \pi/2 \), \( d^2V/d\theta^2 = 4mgk/\pi > 0 \) and these equilibrium points are stable. To find the frequencies of small oscillations about the stable equilibrium points, we use the same formula as the linear case (see pp. 33-34 of the text), except that \( m \) is replaced by \( I \) since this is a rotational problem. Therefore

\[
\omega = \sqrt{\frac{4mgk/\pi}{I}} = \sqrt{\frac{4mgk}{\pi I}}
\]

or

\[
\nu = \frac{\omega}{2\pi} = \frac{1}{\pi} \sqrt{\frac{mgk}{\pi I}}.
\]

3. (a) Since the impulsive force is assumed to act in the \( x \) direction, the final velocity of the mass must be in the \( x \) direction; let it be \( v\hat{x} \). Also, the velocity of the CM of the bar will be in the \( x \) direction; let it be \( V\hat{x} \). Conservation of linear momentum then gives us the relation

\[
mv_0 = mv + MV.
\]
Conservation of energy gives
\[
\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 + \frac{1}{2}I_{CM}\omega^2
\]
where we must include both linear and rotational kinetic energy, \(\omega\) is the angular velocity of the bar about its CM after the collision, and \(I_{CM}\) of a long thin bar is \(ML^2/12\). We have three unknowns (\(v\), \(V\), and \(\omega\)), so we need one more equation. We can use the fact that the same impulsive force that moves the CM of the bar also causes it to rotate. If the impulse is \(F\delta t\), then \(F\delta t = MV\) and the torque about the CM imparted to the bar is \((F\delta t)h = I\omega\). Therefore
\[
MV = F\delta t = \frac{I_{CM}\omega}{h} = \frac{ML^2\omega}{12h}
\]

\[
\omega = \frac{12hV}{L^2}.
\]

Substituting in for \(v\) and \(\omega\) in the energy equation (and multiplying by 2) we get
\[
mv_0^2 = m\left(v_0 - \frac{MV}{m}\right)^2 + MV^2 + \frac{ML^2}{12}\left(\frac{12hV}{L^2}\right)^2.
\]
Solving for \(V\) gives
\[
V = v_0 \frac{2mL^2}{ML^2 + mL^2 + 12mh^2}.
\]
Therefore
\[
v = v_0 - \frac{MV}{m} = v_0 \left[1 - \frac{2ML^2}{ML^2 + mL^2 + 12mh^2}\right]
\]
and
\[
\omega = \frac{24hmv_0}{ML^2 + mL^2 + 12mh^2}.
\]

(b) Since the bar rotates clockwise due to the impulsive force, a point on the opposite end of the bar will tend to move in the negative \(x\) direction due to the rotation (with speed \(\omega(L/2)\)), and will move in the positive \(x\) direction due to the motion of the CM (with speed \(V\)). The net velocity of the point is therefore
\[
v_{net} = V - \omega \frac{L}{2} = \frac{2mLv_0}{ML^2 + mL^2 + 12mh^2} (L - 6h).
\]
This is positive (i.e., the point moves forward immediately after the collision) if \(L > 6h\), or \(h < L/6\).