1. The Doppler shift is given by $\nu = \nu_0(1 + \beta \cos \alpha) / \sqrt{1 - \beta^2}$. Since the velocities involved are small compared to the speed of light, this can be expanded in powers of $\beta$ by using the binomial expansion $(1 - \beta^2)^{-1/2} \approx 1 + \beta^2/2$ to give $\nu \approx \nu_0(1 + \beta \cos \alpha + \beta^2/2)$.

(a) For $v = 12000$ m/s we get $\beta = v/c = 12000/3 \times 10^8 = 0.00004$. If the ship is moving directly away from the Earth $\alpha = \pi$. Since $\beta$ is very small, the $\beta^2$ term is negligible and we get $\nu \approx \nu_0(1 - \beta)$, which gives $\nu = 0.99996 \nu_0 = 9.9996$ MHz. The change in frequency is therefore $-400$ Hz, and the fractional change is $-0.00004$.

(b) In this case $\beta = 1000/3 \times 10^8 = 3.3 \times 10^{-6}$ and $\alpha = \pi/2$. Then $\nu \approx \nu_0(1 + \beta^2/2)$ and the frequency shift is $\Delta \nu = \nu - \nu_0 = \nu_0\beta^2/2 = 5.6 \times 10^{12} \nu_0 = +5.6 \times 10^{-5}$ Hz and the fractional change is $+5.6 \times 10^{-12}$.

(c) Given $\Delta \nu = \nu - \nu_0 = -\beta \nu_0$, the Q value is $Q = |\nu_0/\Delta \nu| = 1/\beta$, or $\beta = 1/Q = 1/200$, so $v = \beta c = 0.005c = 1.5 \times 10^6$ m/s.

2. When approaching the sun, $\alpha = 0$ and $\nu = \nu_0(1 + \beta)/\sqrt{1 - \beta^2}$. Since $v$ is not necessarily small compared to $c$ we do not make any approximations. The wavelength is $\lambda = c/\nu$, so $\lambda = \lambda_0\sqrt{1 - \beta^2}/(1 + \beta)$. The speed that takes $700$ nm to $350$ nm is then given by

$$350 = 700\frac{\sqrt{1 - \beta^2}}{1 + \beta}$$

$$1 + \beta = 2\sqrt{1 - \beta^2}$$

$$5\beta^2 + 2\beta - 3 = 0$$

$$(5\beta - 3)(\beta + 1) = 0$$

This has solutions $\beta = 0.6$ and $-1$; the physically allowed solution is $\beta = 0.6$, so the speed that Doppler shifts the visible into the ultraviolet is $v = 0.6 \, c$.

When moving directly away from the sun, $\alpha = \pi$ and $\nu = \nu_0(1 - \beta)/\sqrt{1 - \beta^2}$ and $\lambda = \lambda_0\sqrt{1 - \beta^2}/(1 - \beta)$. The speed that takes $350$ nm to $700$ nm is then given by

$$700 = 350\frac{\sqrt{1 - \beta^2}}{1 - \beta}$$

$$2(1 - \beta) = \sqrt{1 - \beta^2}$$

$$5\beta^2 - 8\beta + 3 = 0$$

$$(5\beta - 3)(\beta - 1) = 0$$

This has solutions $\beta = 0.6$ and $1$; the physically allowed solution is $\beta = 0.6$, so the speed that Doppler shifts the visible into the infrared is also $v = 0.6 \, c$. 
3. (a) The equation of motion is $\frac{dp}{dt} = F = F_0 \cos \omega t$. Integrating we get $p = (F_0 \sin \omega t)/\omega$, where we have used the initial condition $p_0 = 0$ (since $v_0 = 0$).

(b) We have $m\gamma v = (F_0 \sin \omega t)/\omega$, so using $\gamma = 1/\sqrt{1 - v^2/c^2}$, we get

$$v = \frac{F_0}{m\omega} \frac{\sin \omega t}{\sqrt{1 + \frac{F_0^2}{m^2 \omega^2 c^2} \sin^2 \omega t}}$$

where we have taken the positive root since $v$ and $p$ must be in the same direction.

(c) To find the position, we must integrate the velocity. In the limit that speeds are small, we expand in powers of $c^{-2}$ using the binomial expansion on the square root:

$$v \simeq \frac{F_0}{m\omega} \sin \omega t \left(1 - \frac{F_0^2}{2m^2 \omega^2 c^2} \sin^2 \omega t\right).$$

This is easiest to integrate if we replace $\sin^2 \omega t$ by $1 - \cos^2 \omega t$

$$\frac{dx}{dt} = v = \frac{F_0}{m\omega} \sin \omega t \left[\left(1 - \frac{F_0^2}{2m^2 \omega^2 c^2}\right) + \frac{F_0^2}{2m^2 \omega^2 c^2} \cos^2 \omega t\right]$$

$$x = \frac{F_0}{m\omega^2} \left(1 - \frac{F_0^2}{2m^2 \omega^2 c^2}\right) (1 - \cos \omega t) + \frac{F_0^3}{6m^3 \omega^4 c^2} (1 - \cos^3 \omega t)$$

where we have evaluated both sides between 0 and 1 after integrating.