1. Consider two objects, each with mass $m$, which are attached to springs as shown in the diagram below. The equilibrium lengths of the springs can be taken as zero, and the ends of the springs at $(0, 0)$ and $(a, 0)$ are fixed. Assume that the gravitational field is uniform and acts downward with strength $g$.

(a) Find the equilibrium positions of the two objects. Verify that the equilibrium is stable.

(b) Find the normal mode frequencies and the normal mode coordinates. [Hint: note that the $x$ and $y$ coordinates decouple from each other and can be solved for separately.]

2. Consider two masses that are attached to springs as shown below and are constrained to move in one dimension with no friction. The coordinates $x_1$ and $x_2$ are measured from the equilibrium position, i.e., at $x_1 = 0$ and $x_2 = 0$ all springs are at their equilibrium length. The spring constant $k'$ is much smaller than the other spring constants.

(a) For the unperturbed problem, i.e., in the limit that $k' = 0$, what are the normal mode frequencies and coordinates? [Hint: this should be easy!]

(b) If $k' = \epsilon k$ and $\epsilon \ll 1$, use perturbation theory to find the normal mode frequencies and coordinates to leading order in $\epsilon$. 