1. Th BaBar experiment at the Stanford Linear Accelerator Center (SLAC) collides an electron ($e^-$) head-on with a positron ($e^+$) to create an upsilon particle ($\Upsilon$), which then decays to two $B$ mesons ($B$ and $\bar{B}$). The $\Upsilon$ has mass 10.58 GeV, $B$ mesons have mass 5.28 GeV and the electron and positron can be taken as massless (their actual mass is 0.000511 GeV!), where factors of $c$ are being ignored.

(a) If the electron has energy 9.00 GeV in the lab frame, what must the energy of the positron be in the lab frame so that there is just enough energy in the center of mass to create an $\Upsilon$? If the electron is moving in the $+x$ direction and the positron in the $-x$ direction, what is the velocity of the center of mass relative to the lab frame? [Hint: to find the velocity of the CM, find the Lorentz transformation that makes the spatial component of the total 4-momentum vanish.]

(b) If the $\Upsilon$ decays immediately after being produced, what are the energies and momenta of the $B$ and $\bar{B}$ mesons in the lab frame if they decay (i) in the $+x$ and $-x$ direction in the $\Upsilon$ rest frame, and (ii) in the $+y$ and $-y$ direction in the $\Upsilon$ rest frame?

2. We have seen that in the Compton effect, a photon hits an electron at rest and transfers some of its energy to the electron. In the inverse Compton effect an electron collides with a photon (not at rest!) and changes the direction of the photon.

(a) If an electron with energy $E_1$ moving in the $+x$ direction collides with a photon with energy $E_2$ moving in the $-x$ direction, what are the final energies and momenta of the electron and photon if they are both moving parallel to the $x$ axis after the collision? You may use $m$ as the mass of the electron.

(b) Show that in the example in part (a), the electron and photon simply exchange energies and momenta in the limit that the electron mass can be ignored ($m \ll E_1$).

(c) Show that your result in part (a) for the final photon energy in inverse Compton scattering agrees with the result for Compton scattering when the initial momentum of the electron is set to zero and the Compton scattering angle is set to $\pi$. 