1. [25 points] For each of the following velocity fields, determine if the fluid flow is (i) steady or not, (ii) compressible or incompressible, and (iii) rotational or irrotational. If it is rotational, determine the angular velocity as a function of position; if it is irrotational, determine the velocity potential function.

(a) \( \mathbf{v} = (-2axy, (a - b)y^2, 2byz) \)

(b) \( \mathbf{v} = (2axy + bz^2, ax^2 + 2tyz, 2bxz + ty^2) \)
2. [30 points] Consider a uniform rectangular board of mass $M$, width $a$, length $b$, and negligible thickness lying in the $x$-$y$ plane. One corner of the board (call it point O) is fixed and the board is allowed to freely rotate in its own plane about this point. For this situation:

(a) Sketch the position of the board when it is at rest and in equilibrium. Identify the position of the center of mass and its relationship to the point O.

(b) Prove that the moment of inertia about the point O is $M(a^2 + b^2)/3$. Show your work.

(c) Find the frequency of small oscillations when the board acts as a physical pendulum.

(d) Find the center of oscillation.
3. [20 points] Three forces act on a board lying in the $x$-$y$ plane as shown. Find the equilibrant, i.e., a single additional force and a possible point of action that, when added to the other forces, will keep the board in equilibrium.
4. [25 points] A long, thin board of mass $M$, length $L$ and cross sectional area $A$ is supported at both ends by a vertical force. The ends are otherwise free, i.e., there is no torque applied to the ends and the slope at the ends is not fixed. The two ends of the board are at the same height, $y = 0$. The board material has Young’s Modulus $Y$ and the cross section of the beam has radius of gyration $k$ about a horizontal axis through its centroid. Ignore shear.

(a) Find the function $y(x)$ that describes the position of the board, assuming $x = 0$ at the left end of the board. Also find the support forces at the two ends.

(b) How far below the support points is the middle of the board? What is the magnitude of the slope of the board at the ends?