1. [16 points] A mass $m$ moving in one dimension is connected to a spring with spring constant $k$. The other end of the spring is connected to a point that oscillates according to the equation $x' = B \sin \omega t$. The equilibrium length of the spring is zero, so the potential energy of the spring is $V(x) = k(x - x')^2/2$. There are no other forces acting.

(a) What is the force acting on the mass as a function of time? What is the equation of motion for the mass?

(b) Find the particular solution $x(t)$ for the motion of the mass $m$. 
2. [30 points] A simple pendulum of mass $m$ and constant length $L$ hangs from a point that accelerates horizontally with acceleration $a$ (see figure), The motion of the hanging point is described by $x' = at^2/2$.

(a) Using the angle $\theta$ that the pendulum makes with respect to the vertical (see figure) as the one degree of freedom, what is the Lagrangian of the pendulum?

(b) Find the equation of motion for small angles (i.e., $\theta \ll 1$), and solve for $\theta(t)$ if the initial conditions are $\theta(0) = \theta_0$ and $\dot{\theta}(0) = 0$. 
3. [22 points] A bead of mass $m$ moves without friction on a fixed circular wire of radius $R$ centered at the origin (see figure). Gravity acts in the negative $y$ direction.

(a) Write the kinetic energy in polar coordinates. [Do not impose the constraint on the radius of the ring.]

(b) In what direction does the constraint force act? Label the constraint force $N$. What are the equations of motion for the variables $r$ and $\theta$? [Do not impose the constraint. You do not need to solve the equations.]

(c) Now impose the constraint, find the constraint force $N$, and determine the equation of motion for the remaining degree of freedom. [You do not need to solve the equation of motion.]
4. [16 points] A ring of radius \( R \) and mass \( M \) is centered on the origin in the \( x-y \) plane (see figure).

(a) Find the gravitational potential \( G \) at a point \((0,0,z)\) on the \( z \) axis.

(b) Find the gravitational field \( g \) at the point \((0,0,z)\).

(c) How do \( G \) and \( g \) depend on \( z \) for \( z \gg R \)? Could you have determined them without using your results above? Explain.
5. [16 points] A ball of mass $m$ is thrown vertically down from the top of a tower with initial speed $v_0$. The tower is in the southern hemisphere at latitude $\theta_L$ below the equator. Assume the Earth rotates with angular speed $\omega$, and has radius $R_e$. For this problem, you may ignore the effect of the centrifugal force.

(a) Find the magnitude of the vertical component of the coriolis force. Is it directed up or down? Would your answer be different if the tower was at latitude $\theta_L$ in the northern hemisphere (and if so, how)?

(b) Find the magnitude of the horizontal component of the coriolis force. Is it directed North, East, South, or West? Would your answer be different if the tower was at latitude $\theta_L$ in the northern hemisphere (and if so, how)?