14.9 Molecular hydrogen gas has a molar mass of \(M_{\text{hydrogen}} = 2 \text{ g/mol} = 2 \times 10^{-3} \text{ kg/mol}\). Its rms speed is

\[
\nu_{\text{rms hydrogen}} = \sqrt{\frac{3RT}{M_{\text{hydrogen}}}}.
\]

So

\[
\nu_{\text{rms hydrogen}} = \sqrt{\frac{3(8.315 \text{ J/mol-K})(300 \text{ K})}{2 \times 10^{-3} \text{ kg/mol}}} = 1.03 \times 10^4 \text{ m/s}.
\]

The molar mass of molecular oxygen gas is \(M_{\text{oxygen}} = 32 \text{ g/mol} = 32 \times 10^{-3} \text{ kg/mol}\). Its rms speed is

\[
\nu_{\text{rms oxygen}} = \sqrt{\frac{3RT}{M_{\text{oxygen}}}}.
\]

Divide equation (1) by equation (2).

\[
\frac{\nu_{\text{rms hydrogen}}}{\nu_{\text{rms oxygen}}} = \sqrt{\frac{M_{\text{oxygen}}}{M_{\text{hydrogen}}}} = \sqrt{\frac{32 \times 10^{-3} \text{ kg/mol}}{2 \times 10^{-3} \text{ kg/mol}}} = 4.
\]

So, when at the same temperature,

\[
\nu_{\text{rms hydrogen}} = 4\nu_{\text{rms oxygen}}.
\]

The rms speed of hydrogen gas molecules is 4 times that of oxygen gas molecules at the same temperature.

14.19

a) Let \(n_i\) be the number of particles with speed \(v_i\). The average speed of the collection is, by definition, the weighted average

\[
\langle v \rangle = \frac{\sum n_i v_i}{\sum n_i} = \frac{1}{N} \sum n_i v_i,
\]

which means that

\[
\langle v \rangle = \frac{1}{1010} (40 \times 100 \text{ m/s} + 125 \times 200 \text{ m/s} + 180 \times 300 \text{ m/s} + 215 \times 400 \text{ m/s} + 180 \times 500 \text{ m/s} + 125 \times 600 \text{ m/s} + 75 \times 700 \text{ m/s} + 40 \times 800 \text{ m/s} + 20 \times 900 \text{ m/s} + 10 \times 1000 \text{ m/s}) = 442 \text{ m/s}.
\]

b) The rms speed is defined as

\[
\nu_{\text{rms}} = \sqrt{\frac{\sum n_i v_i^2}{\sum n_i}}.
\]
which means that:

\[ v_{\text{rms}}^2 = \frac{1}{1010} [40 \times (100 \text{ m/s})^2 + 125 \times (200 \text{ m/s})^2 + 180 \times (300 \text{ m/s})^2 + 215 \times (400 \text{ m/s})^2 + 180 \times (500 \text{ m/s})^2 + 125 \times (600 \text{ m/s})^2 + 75 \times (700 \text{ m/s})^2 + 46 \times (800 \text{ m/s})^2 + 20 \times (900 \text{ m/s})^2 + 10 \times (1000 \text{ m/s})^2] \]

so

\[ v_{\text{rms}} = 483 \text{ m/s} . \]

c) The graph appears below:

![Graph showing the distribution of particle speeds.]

d) The temperature of the gas is related to the rms speed by the equation:

\[ v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \Rightarrow T = \frac{Mv_{\text{rms}}^2}{3R} , \]

where \( M \) is the molar mass of the oxygen (O_2) molecules,

\[ M = 2 \times (16 \text{ g/mol}) = 32 \times 10^{-3} \text{ kg/mol} . \]

Putting in the numbers, we have

\[ T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{(32 \times 10^{-3} \text{ kg/mol})(483 \text{ m/s})^2}{3(8.315 \text{ J/mol K})} = 298 \text{ K} . \]
14.30

a) The average kinetic energy of one of the helium atoms is

$$KE_{\text{average}} = \frac{3kT}{2}.$$ 

Putting in the numbers, we have

$$KE_{\text{average}} = \frac{3(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}{2} = 9.21 \times 10^{-21} \text{ J.}$$

b) The mass \( m \) of a single helium atom is

$$m = \frac{4 \times 10^{-2} \text{ kg/mol}}{6.02 \times 10^{23} \text{ particles/mole}} = 6.64 \times 10^{-27} \text{ kg.}$$

The gravitational potential energy of such an atom near the surface of the Earth is

$$PE = -\frac{GM_{\text{Earth}}m}{R_{\text{Earth}}}.$$ 

Putting in the numbers, we find

$$PE = -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(6.64 \times 10^{-27} \text{ kg})}{6.37 \times 10^6 \text{ m}} = -4.16 \times 10^{-19} \text{ J.}$$

c) The ratio is

$$\frac{|PE|}{KE} = \frac{4.16 \times 10^{-19} \text{ J}}{9.21 \times 10^{-21} \text{ J}} = 67.0.$$ 

Despite the large size of this ratio, it is permissible to ignore the gravitational potential energy in kinetic theory because each particle in the gas has essentially the same gravitational potential energy (as long as the vertical extent of the gas is insignificant relative to the radius of the Earth). It is only changes in the gravitational potential energy that are significant, and such changes are insignificant within the volume of the gas.

14.36 Since both the nitrogen and oxygen gases have the same temperature, the average kinetic energy per particle is the same in both. Furthermore, since collisions of the particles with each other and with the walls of the container are assumed elastic, the kinetic energy of the particles in both gases remains constant throughout the mixing process. Thus, in turn, means that the final temperature is the same as the initial temperature. Therefore, before the mixing occurs,

$$P_1V = n_1RT \implies n_1 = \frac{P_1V}{RT} \quad \text{and} \quad P_2V = n_2RT \implies n_2 = \frac{P_2V}{RT}.$$ 

After the two gases have mixed, the final gas also obeys the ideal gas law

$$PV = nRT \implies n = \frac{PV}{RT}.$$ 

However,

$$n = n_1 + n_2 = \frac{P_1V}{RT} + \frac{P_2V}{RT} \implies n = \frac{(P_1 + P_2)V}{RT}.$$ 

Equating the two expressions for \( n \) shows that \( P = P_1 + P_2 \), which is Dalton's law of partial pressures.
14.44

a) The average kinetic energy per particle is

$$\frac{3kT}{2} = \frac{3(1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}{2} = 6.21 \times 10^{-21} \text{ J}.$$  

b) To find the total kinetic energy of all the particles in the gas, we need to know how many there are. So use the ideal gas law to determine the number of moles $n$, use this to find the number of particles and then use the number of particles to find the total kinetic energy:

$$PV = nRT \implies n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \times 10^{-3} m^3)}{(8.315 \text{ J/mol K})(300 \text{ K})} = 4.05 \times 10^{-2} \text{ mol}.$$  

Thus, the number of particles $N$ in the gas is

$$N = nN_A = (4.05 \times 10^{-2} \text{ mol})(6.02 \times 10^{23} \text{ particles/mol}) = 2.44 \times 10^{22} \text{ particles}.$$  

Thus, the internal energy of the whole gas is

$$U = (2.44 \times 10^{22} \text{ particles})(6.21 \times 10^{-21} \text{ J/particle}) = 1.51 \text{ J}.$$  

14.50

a) When you increase the pressure the volume decreases, so the derivative $\frac{dV}{dP}$ is negative which, when combined with the minus sign, makes the right hand side, and thus $\kappa$, positive.

b) In order to evaluate the compressibility $\kappa$, we need to express the volume as a function of pressure. To do this, use the ideal gas law,

$$PV = nRT \implies V = \frac{nRT}{P}.$$  

At constant $T$, the derivative of $V$ with respect to $P$ is

$$\frac{dV}{dP} = nRT \left(\frac{-1}{P^2}\right) = PV \left(\frac{-1}{P}\right) = \frac{-V}{P}.$$  

Therefore,

$$\kappa_{\text{isothermal}} = -\frac{1}{V} \frac{dV}{dP} = -\left(\frac{1}{V}\right) \left(\frac{-V}{P}\right) = \frac{1}{P}.$$  

c) For an adiabatic process,

$$PV^n = \text{constant} \implies V = \left(\frac{\text{constant}}{P}\right)^{1/n} = (\text{constant})^{1/n} P^{-1/n}.$$  

Therefore,

$$\frac{dV}{dP} = (\text{constant})^{1/n} \left(\frac{-1}{P}\right) P^{-1/n-1} = P^{1/n} V \left(\frac{-1}{P}\right) P^{-1/n-1} = \frac{V}{\gamma P},$$  

and

$$\kappa_{\text{adiabatic}} = \frac{1}{V} \left(\frac{-V}{\gamma P}\right) = \frac{1}{\gamma P}.$$
Combining the answer above with the answer to b), we have
\[
\frac{1}{\gamma\text{adiabatic}} = \frac{1}{\gamma\text{isothermal}} = \frac{1}{\gamma} \left( \frac{1}{\gamma} \right) = \frac{1}{\gamma^2}
\]
which means that
\[
\gamma\text{adiabatic} = \gamma\text{isothermal}.
\]

14.52
a) Since the gas is compressed isothermally, the temperature remains constant throughout the entire process. However, for a fixed quantity of an ideal gas, the internal energy is only a function of its temperature. Therefore, since the temperature does not change, the internal energy of the gas also does not change, and
\[
\Delta U = 0 \text{ J}.
\]
b) The quantity \( W \) is the work done by the gas, which is the negative of the work done by you on the gas (because of Newton's third law). Hence, using this information in the first law of thermodynamics, we have
\[
Q = \Delta U + W \implies Q = 0 \text{ J} + (-500 \text{ J}) = -500 \text{ J}.
\]
So the heat transfer to the gas is -500 J. Note, however, that the minus means the heat transfer of 500 J really is from the gas.

14.56
a) An isobaric expansion is at constant pressure, so it is represented as a horizontal line on a \( P-V \) diagram. An isothermal expansion is at constant temperature, so it follows an isotherm. An adiabatic expansion lowers the temperature, so it is below the isothermal process on the \( P-V \) diagram.

b) The work done by the gas in each process is the area under the appropriate curve on the \( P-V \) diagram. The isobaric process has the greatest area under the curve, so the work done by the gas during the isobaric process is greater than that done in the other two processes.

c) There is no heat transfer to the gas during an adiabatic process. For the isothermal process, there is no change in the internal energy since the internal energy of an ideal gas is a function only of the absolute temperature, and that does not change in the isothermal expansion. Hence, we see from the first law of thermodynamics, \( Q = \Delta U + W \), that the heat transfer to the gas during any isothermal expansion is equal to the work done by the gas. On the other hand, during the isobaric expansion, maximum work is done and the temperature of the gas increases, so \( Q \) is maximized. Hence the heat transfer to the gas is greatest in the isobaric process.

d) The process with the greatest change in temperature will have the greatest change in the internal energy. The isothermal process has no change in the temperature and, as a result, no change in the internal energy. For the adiabatic process, the gas cools, so the change in its internal energy is negative. On the other hand, for the isobaric process, the gas increases its temperature and so has a positive change in its internal energy. Hence the isobaric process has the greatest change in the internal energy of the gas.
14.58 In an adiabatic process, the pressure and volume satisfy the relation

\[ PV^\gamma = \text{constant} \implies P = \frac{\text{constant}}{V^\gamma}. \]

The work done by the gas is defined as

\[ W = \int_{V_i}^{V_f} P \, dV. \]

Putting the above expression for \( P \) into this integral, we have

\[ W = \int_{V_i}^{V_f} \frac{\text{constant}}{V^\gamma} \, dV = \int_{V_i}^{V_f} \left( \frac{\text{constant}}{V^\gamma} \right) \frac{dV}{V^\gamma}. \]

\[ = \left( \frac{\text{constant}}{\gamma + 1} \right) \left( V_f^{\gamma+1} - V_i^{\gamma+1} \right) \]

\[ = \left( \frac{\text{constant}}{V_i^{\gamma+1}} \right) \left( \frac{V_f}{\gamma + 1} \right) - \left( \frac{\text{constant}}{V_i^{\gamma+1}} \right) \left( \frac{V_i}{\gamma + 1} \right) \]

\[ = P_i \left( \frac{V_f}{\gamma + 1} \right) - R \left( \frac{V_i}{\gamma + 1} \right) \]

\[ = \frac{1}{\gamma - 1} (P_i V_f - R V_i). \]