**Thermodynamics Worksheet**

For an ideal gas, the internal energy $U$ is directly proportional to the temperature $T$. (This is because the internal energy is just the total kinetic energy of all of the gas molecules, and the temperature is defined to be equal to the average molecular kinetic energy.) For a monatomic ideal gas, the relationship is given by $U = \frac{3}{2} nRT$, where $n$ is the number of moles of gas, and $R$ is the universal gas constant.

1. Find a relationship between the internal energy of $n$ moles of ideal gas, and pressure and volume of the gas. Does the relationship change when the number of moles is varied?

2. Suppose that $m$ moles of an ideal gas are contained inside a cylinder with a movable piston (so the volume can vary). At some initial time, the gas is in state $A$ as shown on the $PV$-diagram in Figure 1. A thermodynamic process is carried out and the gas eventually ends up in State $B$. Is the internal energy of the gas in State $B$ greater than, less than, or equal to its internal energy in State $A$? (That is, how does $U_B$ compare to $U_A$?) Explain.

![Figure 1](image)

3. If a system starts with an initial internal energy of $U_{initial}$ and ends up with $U_{final}$ some time later, we symbolize the change in the system’s internal energy by $\Delta U$ and define it as follows: $\Delta U = U_{final} - U_{initial}$.
   a. For the process described in #2 (where the system goes from State $A$ to State $B$), is $\Delta U$ for the gas system greater than zero, equal to zero, or less than zero?
   b. During this process, was there any energy transfer between the gas system and its surrounding environment? Explain.
Energy Transfer

There are two separate methods by which energy can be transferred into or out of a gas system:

A. The gas can be held in a rigid container with fixed volume, where the container is made of a material that is a thermal conductor such as metal. Then the container can be heated (for instance, with a flame) or cooled (for instance, by putting it into ice water). If the ideal gas temperature changes, we know that its internal energy must be changing as well. The symbol $Q$ represents the amount of energy transferred to the system when the system’s volume is unchanging and there is a temperature difference between system and surroundings. **Note:** $Q$ may be positive or negative. ($Q$ is also called “heat,” “heat added,” or “heat transfer.”)

B. The gas can be held in an insulated container made of a material that has zero thermal conductivity. In this case, if the volume of the container is not changed, the gas will not experience any change in temperature regardless of the temperature of the surroundings. However, if the volume of the container can be changed – for instance, if the container is a cylinder with a movable piston – then experiments show that the temperature of the gas will change when the volume changes. In fact, experiments done with a gas enclosed in a thermally insulated container show that:

i. if the gas is compressed so its volume decreases, the gas temperature increases. (Therefore, the internal energy of the system is increasing.)

ii. if the gas expands so its volume increases, the gas temperature decreases. (Therefore, the internal energy of the system is decreasing.)

If the gas expands, we say that the gas does positive work “$W$” on the surrounding environment, and so $W$ is greater than zero. If the gas is compressed, we say that the surrounding environment does positive work on the gas and so $W$ is less than zero. [The symbol “$W$” is defined to be the positive work done by the system on the surrounding environment.]

4. An ideal gas is held in a **rigid container**. For the following cases, state whether $Q$ is greater than zero, equal to zero, or less than zero; also answer the same question for $\Delta U$.

   a. the temperature of the gas increases:
      
      $Q$: ________________
      
      $\Delta U$: ________________
      
   b. the temperature of the gas decreases:
      
      $Q$: ________________
      
      $\Delta U$: ________________
      
   c. the temperature of the gas remains unchanged:
      
      $Q$: ________________
      
      $\Delta U$: ________________
5. An ideal gas is held in a \textit{thermally insulated, flexible container} where the volume can vary. For the following cases, state whether \( W \) is greater than zero, equal to zero, or less than zero; also answer the same question for \( \Delta U \).

\begin{align*}
\text{a. the temperature of the gas increases:} & \\
W & : \underline{\text{___________}} \\
\Delta U & : \underline{\text{___________}} \\
\text{b. the temperature of the gas decreases:} & \\
W & : \underline{\text{___________}} \\
\Delta U & : \underline{\text{___________}} \\
\text{c. the temperature of the gas remains unchanged:} & \\
W & : \underline{\text{___________}} \\
\Delta U & : \underline{\text{___________}} \\
\end{align*}

6. According to the principle of conservation of energy, any change in a system’s energy must be due to energy that is transferred to the system from its environment, or away from the system to the environment. If a gas system is held in a container that is both thermally \textit{conducting} and can change its volume, energy transfer can take place both through heat transfer \( Q \) and work \( W \). Write an expression relating \( \Delta U \) (the change in a system’s internal energy) to \( Q \) and \( W \):

\[ \Delta U = \underline{\text{___________}} \]

\textit{Check} your expression by testing it with the answers you have given for \#4 and \#5. (\textit{Hint}: What is the value of \( W \) for the cases in \#4? What is the value of \( Q \) for the cases in \#5?)

The \( PV \)-diagram in Figure 2 shows an initial state (“\( i \)”), a final state (“\( f \)”), and four other states (\( A, B, C, \) and \( D \)). A fixed quantity of ideal gas carries out two different processes:

\begin{align*}
\text{a. Process \#1: } i & \rightarrow A \rightarrow B \rightarrow f \\
\text{b. Process \#2: } i & \rightarrow C \rightarrow D \rightarrow f \\
\end{align*}

We will be considering the following quantities:

\begin{align*}
\Delta U_1 & : \text{the change in the system’s internal energy during Process \#1.} \\
\Delta U_2 & : \text{the change in the system’s internal energy during Process \#2.} \\
Q_1 & : \text{the heat transferred to the system during Process \#1.} \\
Q_2 & : \text{the heat transferred to the system during Process \#2.} \\
W_1 & : \text{the work done by the system during Process \#1.} \\
W_2 & : \text{the work done by the system during Process \#2.} \\
\end{align*}

We will also refer to sub-processes, such as \( i \rightarrow A \) and \( D \rightarrow f \), etc.
7. Rank the temperature of the gas at the six points $i$, $A$, $B$, $C$, $D$, and $f$. (Remember this is an ideal gas.)

8. Consider all sub-processes represented by straight-line segments. For each one, state whether the work done by the gas is positive, negative, or zero. In the second column, rank all six processes according to their $\Delta U$. (Pay attention to the sign of $\Delta U$.) If two segments have the same $\Delta U$, give them the same rank. In the last column, state whether heat is transferred to the gas, transferred away from the gas, or is zero (i.e., no heat transfer). **Hint:** First determine $U$ for each point using the result of #1 on page 1. You can choose any numbers for the $P$ and $V$ scales; answers won’t change!

<table>
<thead>
<tr>
<th>Process</th>
<th>Is $W$, $+, -, 0$?</th>
<th>rank according to $\Delta U$</th>
<th>heat transferred to, transferred away, or zero?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \rightarrow A$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A \rightarrow B$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B \rightarrow f$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i \rightarrow C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C \rightarrow D$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D \rightarrow f$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Consider only the sub-processes that have $W = 0$. Of these, which has the greatest absolute value of heat transfer $Q$? Which has the smallest absolute value of $Q$?

10. Rank the six segments in the table above according to the absolute value of their $W$. **Hint:** For processes at constant pressure, $W = P \Delta V$.

11. Using your answers to #8 and #10, explain whether $W_1$ is greater than, less than, or equal to $W_2$. [Refer to definitions, page 3.] Is there also a way to answer this question using an “area” argument?

12. Is $Q_1$ greater than, less than, or equal to $Q_2$? Explain. **Hint:** Compare the magnitude of $\Delta U_1$ and $\Delta U_2$, and make use of the answer to #6.