STRESS, STRAIN & ELASTIC MODULUS

\[
\frac{\text{STRESS}}{\text{STRAIN}} = \text{ELASTIC MODULUS (Hooke's Law)}
\]

TENSILE STRESS = \( \frac{F}{A} \)

1 pascal = 1 Pa = 1 N/m²
\( \rightarrow 1.450 \times 10^{-4} \) psi

[1 psi = 6895 Pa]

TENSILE STRAIN = \( \frac{\Delta L - L_0}{L_0} \) or \( \frac{L - L_0}{L_0} \)

\[ = \frac{\Delta L}{L_0} \text{ or } \frac{\Delta L}{L_0} \text{ STRETCH/UNIT LENGTH} \]
STRESS, STRAIN & ELASTIC MODULI - 2

YOUNG'S MODULUS $Y$ or $E$

$Y = E = \frac{\text{TENSILE STRESS}}{\text{TENSILE STRAIN}}$

$Y = \frac{F_L/A}{\Delta l/l_0} = \frac{F_L}{A \Delta l}$
A solid rectangular parallelepiped.

A shear stress applied to a solid material.
SHEAR STRESS & STRAIN

SHEAR STRESS \[ \sigma = \frac{F_{ii}}{A} \]

SHEAR STRAIN \[ \varepsilon = \frac{x}{h} = \tan \phi \]

SHEAR MODULUS (S or G)

\[ S = \frac{\text{SHEAR STRESS}}{\text{SHEAR STRAIN}} = \frac{F_{ii}/A}{x/h} \]

\[ = \frac{F_{ii}}{A} \frac{h}{x} = \frac{F_{ii}/A}{\phi} \]

\[ = C = \frac{F_{ii}/A}{\Delta R/R} \]
BULK STRESS & STRAIN

BULK (Volume) STRESS
BULK (Volume) STRAIN

NOTE: IN LIQUID: \( \rho = \frac{F_L}{A} \) \( L \) pressure

NOTE: 1 atmosphere = 1 atm

\( 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 14.7 \text{ lb/in}^2 \)

BULK STRESS \( \sigma = \frac{\Delta P}{A} \)

BULK STRAIN \( \varepsilon = \frac{\Delta V}{V_0} \) \{ LINEAR ANALOGY \}

BULK (ELASTIC) MODULUS \( \frac{\sigma}{\varepsilon} = \frac{\text{STRESS}}{\text{STRAIN}} \)

\( B = -\frac{\Delta P}{\Delta V/V_0} \)

COMPRRESSIBILITY \( c = \frac{1}{k} \)

\( k = -\frac{1}{V_0} \frac{\Delta V}{\Delta P} \)
Ch 11- Prob 1 A vertical steel wire with a diameter of 1.0 mm is 2.0 m long. What mass should be hung from the wire to stretch it 1.0 m?

Note: Young's modulus for steel = \(20 \times 10^9\ \text{N/m}^2\).

Young's modulus \(E = \frac{\text{stress}}{\text{strain}}\)

\[
E = \frac{F A}{\Delta l} \Rightarrow F = \frac{E A \Delta l}{l} = m g
\]

\[m = \frac{E A \Delta l}{g l} = \frac{E \pi r^2 \Delta l}{g l} = \frac{(20 \times 10^9\ \text{N/m}^2) \pi (1 \times 10^{-3} \text{m})^2}{(2\text{.0 m})} \]

\[m = \frac{(9.8 \text{m/s}^2) (2\text{.0 m})}{9.81 \text{m/s}^2} \]

\[m = 8.0 \text{ kg}\]

\[
\text{Unit: } \frac{(\text{N/m}) \ (\text{m}^2) (\text{m})}{\text{(m/s}^2) (\text{m})} = \frac{\text{N m}^{-1}}{\text{m}^2} = \frac{\text{Ns}^2}{\text{m}^2} = (\frac{\text{kg m}}{\text{s}^2}) \frac{\text{s}^2}{\text{m}^2} = \text{ kg}
\]
Ch 11 - Prob 2

A stool in a diner is supported by a solid steel column of diameter 4.0 cm and length 6.5 cm. When a 90 kg trucker sits on the chair with his feet off the floor, determine the compression of the column. Neglect the mass of the seat of the stool.

Note: Young's modulus for steel = $20 \times 10^{10}$ N/m$^2$

Young's modulus: $E = \frac{\sigma}{\epsilon}$

$\sigma = \frac{F}{A}$

$\epsilon = \frac{\Delta x}{x}$

$\Delta x = \frac{F \epsilon}{AE}$

Where the magnitude of the applied force is the weight of the trucker (Problem assumption: Neglect the mass of the seat of the stool)

And the only area of compression is the cross-sectional area of the stool's support column $\Rightarrow \% E = E(steel) = 20 \times 10^{10}$ N/m$^2$

$\Delta l = \frac{F \epsilon}{AE} = \frac{m(trucker) g l}{\pi r^2 E(steel)}$

$\Delta l = \frac{90 \text{ kg}}{\pi (2.0 \times 10^{-2})^2 (20 \times 10^{10} \text{ N/m}^2)}$

$\Delta l = 2.3 \times 10^{-6} \text{ m} = 2.3 \text{ microns}$
Ch. 11 - Prob 5  A 2.00 m long bungee cord of 8.0 mm diameter stretches 250 cm when an 80 kg college-level turkey is suspended from a hook on one end of the cord. Find Young's modulus for the cord.

Assume: the magnitude of the applied force on the line is the magnitude of the student's weight. \[ \Rightarrow \]

Assume - Cord is massless \[ \Rightarrow \] Ignore both cord mass and weight.

The area of cord \[ \Rightarrow \] Calculating cord's \[ \Rightarrow \] assumed constant diameter (as given).

At all times and all tensions.

\[
\frac{E}{d} = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/l} = \frac{F \times l}{A \Delta l} = \frac{Mg \times l}{\pi r^2 \Delta l}
\]

\[
E = \frac{(80 \text{ kg})(9.81 \text{ m/s}^2)(2.00 \text{ m})}{\pi (4.0 \times 10^{-3} \text{ m})^2 (2.50 \text{ m})}
\]

\[ E = 1.2 \times 10^7 \text{ N/m}^2 \]
Chill Prob 8 WHAT PRESSURE WILL DECREASE THE VOLUME OF A SAMPLE OF WATER BY 1.0%.

The Bulk Modulus \( \beta = \frac{\text{Pressure}}{\text{Volume Strain}} = -\frac{P}{\frac{AV}{V}} \)

\[ \Rightarrow P = -\frac{\beta AV}{V} \]

Since the volume decreases by 1.0%,

The Volume Strain \( \frac{AV}{V} = -0.010 \)

The Bulk Modulus of \( H_2O \) (Liq) \( = 0.20 \times 10^{10} \text{ Pa} \)

Hence:

\[ P = -(0.20 \times 10^{10}\text{ Pa})(-0.010) = 2.0 \times 10^7 \text{ Pa} \]

This means that a pressure of about 200 times atmospheric pressure is needed to decrease the volume of a sample of water by 1%.

This is why for most purposes water may be considered an incompressible fluid.