PROBLEM SOLUTION METHOD – Suggested Approach

There are a number of possible methods for organizing and solving problems or answering questions where relative quantitative or explicit mathematical (or numeric) results are required. Not all problems require such a detailed approach, but for understanding results and organization for study, the following problem solving method is suggested. An example problem solution is attached. (You will be responsible for this problem in the appropriate test.)

1) Read the problem carefully and completely.
   A) Determine what the problem is asking.
   B) Determine what information is given.
   C) Note what information or facts are not given.

2) Draw a figure, label figure and define parameters labeled in figure.

3) List assumptions made – why and the consequences of such assumptions (limits of validity).

4) Note or list constants and/or constraints applied to system due to the assumptions chosen and/or the figure.

5) Determine what physical principle(s) apply. (NOTE: In the textbook used all problems are listed in subsections based in those used in the chapter.)
   A) Basic laws used.
   B) Physical formulas used. (label in margin for reference)
   C) Approximations based on assumptions.

6. Show algebraic simplification and/or solutions(s).

7. Put in numeric values and units to produce quantitative solution (if required).

8. Check results for both numeric and unit correctness (order of magnitude correctness and units compatibility most important). NOTE: Some solutions because of assumptions and/or approximations may be only order of magnitude or what is known as back of the envelope calculations.

9. NOTE: One may do the following to simplify problems.
   A) Number equations and refer to these numbers in sequencing solutions.
   B) Box solutions (in red or contrasting color).
   C) Do formal detailed numeric checks.

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167. A pipe of length $L = 25.0\, \text{m}$ that is open at one end contains air at atmospheric pressure. It is thrust vertically into a freshwater lake until the water rises halfway up in the pipe, as shown in Fig. 20-20. What is the depth $h$ of the lower end of the pipe? Assume that the temperature is the same everywhere and does not change.

**Given:**
1. Pressure in pipe = 1 atm before thrust into $\text{H}_2\text{O}$.
2. Pipe length $L = 25.0\, \text{m}$
3. Pipe closed top end (see figure)
4. $\text{H}_2\text{O}$ in pipe rises half way up pipe ($1/2$ air + $1/2$ $\text{H}_2\text{O}$)
5. $h =$ depth of pipe bottom from $\text{H}_2\text{O}$ surface

**Find:** $h =$ depth of pipe bottom from $\text{H}_2\text{O}$ surface

**Assumptions:**
1. Location is approx sea level or 1 atmosphere in pressure
2. Pipe cross sectional area ($A$) is defined as constant
3. Pipe’s closed end is still above $\text{H}_2\text{O}$ surface $ightarrow$ buoyant: $h < L$
4. Density of $\text{H}_2\text{O}$: $\rho_{\text{H}_2\text{O}} = \rho$
   - constant with depth (not exactly true even at 25 m depth)
(a) The atmosphere modeled by the ideal gas law

(b) Air, H₂O, and materials all at the same temperature.

**Solution:** Ideal gas law \( PV = nRT \) \[\text{Eq. 20.5}\]

Given that \( T = \text{const} \) \( \Rightarrow \)

\( PV = \text{constant} \)

First, when the open end is above the H₂O surface in the air (just above H₂O surface), pressure in pipe: \( P = 1 \text{ atm} \).

Since \( PV = \text{const} \) when gas is compressed to \( \frac{1}{2} \) the volume \( (V = \frac{1}{2}A) \) \( A = \text{const} \) \( \Rightarrow \)

\( \frac{1}{2}V \rightarrow \frac{1}{2}L \rightarrow \frac{1}{2} \) (see figure)

If \( P(1 \text{ atm})V = \frac{1}{2}P \Rightarrow \)

\( P_2 = 2P_1 = 2 \text{atm} \)

The extra pressure \( (1 \text{ atm}) \) on the surface of H₂O balances force \( Mg \) of the displaced H₂O

\[ M = \rho V \]

\[ \rho = \text{density of H₂O} \]

\[ V = \text{volume of displaced H₂O} \]

\[ \Delta PA = Mg = \rho Vg = \rho \left[A \left(h - \frac{1}{2}\right)\right] g \]

**Note:** \( \int Ah = \text{volume of pipe below surface} \)

\( A/2 = \text{volume of H₂O in pipe not displaced} \)

\( A\left(h - \frac{1}{2}\right) = \text{volume of H₂O in pipe displaced} \)
Solve equation:

\[ \Delta P = \rho \left[ A \left( h - \frac{L}{2} \right) \right] g \]
\[ \frac{\Delta P}{\rho g} = A \left( h - \frac{L}{2} \right) \]
\[ h - \frac{L}{2} = \frac{\Delta P}{A \rho g} \]
\[ h = \frac{\Delta P}{\rho g} + \frac{L}{2} \]

**NOTE:**
1. \( \Delta P = P_2 - P_1 \)
   \[ = 2 \text{ atm} - 1 \text{ atm} \]

**Numeric solution:**

\[ h = \frac{-1 \times 10^5 \text{ Pa}}{(1 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} + \frac{25 \text{ m}}{2} \]

\[ = 10.3 \text{ m} + 12.5 \text{ m} \]
\[ = 22.8 \text{ m} \]

**CHECK:**

1. \( h < L \) or
2. UNITS CHECK.
   \[ \frac{\text{Pa}}{\text{kg/m}^3 \text{m}^2} \left( \frac{\text{kg}}{\text{m} \text{s}^2} \right) \left( \frac{\text{m}}{\text{s}^2} \right) = \frac{\text{N/m}^2}{\text{m}^2} = \text{m} \]

**NOTE:**
1. \( 1 \text{ Pa} = 1 \text{ N/m}^2 \)
2. \( 1 \text{ kg/m} \text{sec}^2 = 1 \text{ N} \)
3. \( \frac{\text{N/m}^2}{\text{m}^3} = \text{m} \)