Basics of Heat Transfer
In the simplest of terms, the discipline of heat transfer is concerned with only two things: temperature, and the flow of heat. Temperature represents the amount of thermal energy available, whereas heat flow represents the movement of thermal energy from place to place.

On a microscopic scale, thermal energy is related to the kinetic energy of molecules. The greater a material’s temperature, the greater the thermal agitation of its constituent molecules (manifested both in linear motion and vibrational modes). It is natural for regions containing greater molecular kinetic energy to pass this energy to regions with less kinetic energy.

Several material properties serve to modulate the heat transfered between two regions at differing temperatures. Examples include thermal conductivities, specific heats, material densities, fluid velocities, fluid viscosities, surface emissivities, and more. Taken together, these properties serve to make the solution of many heat transfer problems an involved process.

Heat Transfer Mechanisms
Heat transfer mechanisms can be grouped into 3 broad categories:

**Conduction:** Regions with greater molecular kinetic energy will pass their thermal energy to regions with less molecular energy through direct molecular collisions, a process known as conduction. In metals, a significant portion of the transported thermal energy is also carried by conduction-band electrons.

**Convection:** When heat conducts into a static fluid it leads to a local volumetric expansion. As a result of gravity-induced pressure gradients, the expanded fluid parcel becomes buoyant and displaces, thereby transporting heat by fluid motion (i.e. convection) in addition to conduction. Such heat-induced fluid motion in initially static fluids is known as free convection.

For cases where the fluid is already in motion, heat conducted into the fluid will be transported away chiefly by fluid convection. These cases, known as forced convection, require a pressure gradient to drive the fluid motion, as opposed to a gravity gradient to induce motion through buoyancy.

**Radiation:** All materials radiate thermal energy in amounts determined by their temperature, where the energy is carried by photons of light in the infrared and visible portions of the electromagnetic spectrum. When temperatures are uniform, the radiative flux between objects is in equilibrium and no net thermal energy is exchanged. The balance is upset when temperatures are not uniform, and thermal energy is transported from surfaces of higher to surfaces of lower temperature.

Fourier Law of Heat Conduction
When there exists a temperature gradient within a body, heat energy will flow from the region of high temperature to the region of low temperature. This phenomenon is known as conduction heat transfer, and is described by Fourier's Law (named after the French physicist Joseph Fourier),

\[ \dot{Q} = -k \left( \frac{dT}{dx} \hat{i} + \frac{dT}{dy} \hat{j} + \frac{dT}{dz} \hat{k} \right) \]
This equation determines the heat flux vector \( \vec{q} \) for a given temperature profile \( T \) and thermal conductivity \( k \). The minus sign ensures that heat flows down the temperature gradient.

**Heat Equation (Temperature Determination)**

The temperature profile within a body depends upon the rate of its internally-generated heat, its capacity to store some of this heat, and its rate of thermal conduction to its boundaries (where the heat is transferred to the surrounding environment). Mathematically this is stated by the **Heat Equation**,

\[
\left( \frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} \right) - \frac{1}{\alpha} \frac{dT}{dt} = -\frac{1}{k} q_{\text{gen}}
\]

In the Heat Equation, the **power generated per unit volume** is expressed by \( q_{\text{gen}} \). The **thermal diffusivity** \( \alpha \) is related to the thermal conductivity \( k \), the specific heat \( c \), and the density \( \rho \) by,

\[
\alpha = \frac{k}{\rho c}
\]

For **Steady State** problems, the Heat Equation simplifies to,

\[
\left( \frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{d^2T}{dz^2} \right) = -\frac{1}{k} q_{\text{gen}}
\]

**Derivation of the Heat Equation**

The heat equation follows from the **conservation of energy** for a small element within the body,

\[
\text{heat conducted in} + \text{heat generated within} = \text{heat conducted out} + \text{change in energy stored within}
\]

We can combine the heats conducted in and out into one "net heat conducted out" term to give,

\[
\text{net heat conducted out} = \text{heat generated within} - \text{change in energy stored within}
\]

The change in internal energy \( U \) is related to the body's ability to store heat by raising its temperature, given by,

\[
\frac{dU}{dt} = \rho c \frac{dT}{dt}
\]

**Steady State 1-Dimensional Heat Conduction**
For problems where the temperature variation is only 1-dimensional (say, along the $x$-coordinate direction), Fourier's Law of heat conduction simplifies to the scalar equations,

\[ q = -k \frac{dT}{dx} \quad \text{and} \quad \frac{dQ}{dt} = -kA \frac{dT}{dx} \]

where the heat flux $q$ depends on a given temperature profile $T$ and thermal conductivity $k$. The minus sign ensures that heat flows down the temperature gradient.

In the above equation on the right, $\frac{dQ}{dt}$ represents the heat flow through a defined cross-sectional area $A$, measured in watts,

\[ \frac{dQ}{dt} = \int_A P \, dA \]

Integrating the 1D heat flow equation through a material's thickness $\Delta x$ gives,

\[ \frac{dQ}{dt} = -\frac{kA}{\Delta x} (T_2 - T_1) \]

where $T_1$ and $T_2$ are the temperatures at the two boundaries.

**The R-Value in Insulation**

In general terms, heat transfer is quantified by Newton's Law of Cooling,

\[ \frac{dQ}{dt} = hA \Delta T \]

where $h$ is the heat transfer coefficient. For conduction, $h$ is a function of the thermal conductivity and the material thickness,

\[ h = \frac{k}{\Delta x} \]

In words, $h$ represents the heat flow per unit area per unit temperature difference. The larger $h$ is, the larger the heat transfer $Q$.

The inverse of $h$ is commonly defined as the $R$-value,

\[ R = \frac{1}{h} = \frac{\Delta x}{k} \]

The $R$-value is used to describe the effectiveness of insulations, since as the inverse of $h$, it represents the resistance to heat flow. The larger the $R$, the less the heat flow $\frac{dQ}{dt}$.

$R$ is often expressed in imperial units when listed in tables.

To convert $R$ into a thermal conductivity $k$, we must divide the thickness of the insulation by the $R$ value (or just solve for $k$ from the above equation),
By comparing the steady state heat flow equation with Ohm's Law for current flow through a resistor, we see that they have similar forms,

\[ \frac{dQ}{dt} = -\frac{kA}{\Delta x} \Delta T \quad \Leftrightarrow \quad I = \frac{1}{R} \Delta V \]

We can therefore draw the following analogies:

<table>
<thead>
<tr>
<th>Heat Flow, ( dQ/dt )</th>
<th>Current, ( I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature Difference, ( T_1 - T_2 )</td>
<td>Voltage Difference, ( V_1 - V_2 )</td>
</tr>
<tr>
<td>Thermal Resistance, ( R_T = \Delta x/k*A )</td>
<td>Electrical Resistance, ( R )</td>
</tr>
</tbody>
</table>

The electrical to heat conduction analogy allows one to apply laws from circuit theory to solve more complicated conduction problems, such as the heat flow through conducting layers attached in parallel or series.