Ch. 20 Study Problems

Week of December 6, 2004


Q20.1

A pot is half-filled with water, and a lid is placed on the pot, forming a tight seal so that no water vapor can escape. The pot is heated on a stove, forming water vapor inside the pot. The heat is then turned off and the water vapor condenses back to liquid. Is this cycle reversible or irreversible?

As long as this cycle is carried out slowly enough that the temperature of the water and the vapor are always at nearly the same uniform temperature, the cycle involving the liquid water and vapor is reversible. The reason is that the cycle can be reversed at any point of the warming or cooling parts of the cycle by reducing or increasing the heat input to the pot, respectively.

Q20.13

Suppose you want to increase the efficiency of a heat engine. Would it be better to increase $T_H$ or to decrease $T_C$ by an equal amount? Why?

The initial maximum (ideal) efficiency (the Carnot efficiency) of a reversible heat engine operating between two temperature reservoirs is

$$e_{\text{initial}} = 1 - \frac{T_C}{T_H}.$$ 

If $T_H$ is increased by an amount $\Delta T$, then

$$e = 1 - \frac{T_C}{T_H + \Delta T} = 1 - \frac{T_C}{T_H(1 + \Delta T/T_H)}.$$ 

Now using the Tailor series expansion $(1 + x)^n = 1 + n x + \ldots$ with $n = -1$ and assuming that $x = \Delta T/T_H \ll 1$, one gets

$$e \approx 1 - \frac{T_C}{T_H} \left(1 - \frac{\Delta T}{T_H}\right) = 1 - \frac{T_C}{T_H} + \frac{T_C \Delta T}{T_H^2} = e_{\text{initial}} + \frac{T_C \Delta T}{T_H^2}.$$ (1)

On the other hand, if $T_C$ is decreased by $\Delta T$, then

$$e = 1 - \frac{T_C - \Delta T}{T_H} = 1 - \frac{T_C}{T_H} + \frac{\Delta T}{T_H} = e_{\text{initial}} + \frac{\Delta T}{T_H}.$$ (2)

Comparing equations (1) and (2), one sees that a larger $e$ is obtained by decreasing the temperature of the cold reservoir, because in equation (1) $T_C/T_H < 1$. 

Q20.15

Real heat engines, like the gasoline engine in a car, always have some friction between their moving parts, although lubricants keep the friction to a minimum. Would a heat engine with completely frictionless parts be 100% efficient? Why or why not? Does the answer depend on whether or not the engine runs on the Carnot cycle? Again, why or why not?

The definition of the thermal efficiency of a heat engine is

\[ e = \frac{W}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|} . \]

In order for \( e = 1 \), one would need the thermal energy \( U \) contained in the exhaust gas to be \( U = Q_C = 0 \). The thermal energy of an ideal gas is, apart from a possible additive constant, given by \( U = n C_V T \) where \( T \) is the absolute temperature in Kelvins. Thus in order for \( Q_C \) to be zero, the temperature of the cold reservoir \( T_C = 0 \) K. It is not possible theoretically for the cold reservoir to be at exactly zero Kelvin. This discussion applies to any type of cycle that the engine runs on including the Carnot cycle.

Thus it is impossible for any heat engine to be exactly 100% efficient. This is one argument for the engine statement of the 2nd law of thermodynamics which states that it is not possible to completely covert thermal energy into work using a heat engine (which operates in cycles).

One should note, however, that for a thermodynamic process that is not a cycle, it is possible to completely covert thermal energy into work. This occurs during a reversible isothermal expansion of an ideal gas, where \( \Delta U = 0 \) and hence from the first law of thermodynamics the heat absorbed by the system \( Q \) is completely converted into work \( W \): \( Q = W \). To be truly reversible, however, the process would require an infinite amount of time to carry out.

20.1

A diesel engine performs 2200 J of mechanical work and discards 4300 J of heat each cycle. (a) How much heat must be supplied to the engine in each cycle? (b) What is the thermal efficiency of the engine?

(a) \(|Q_H| = |W| + |Q_C| = 2200 \text{ J} + 4300 \text{ J} = 6500 \text{ J} .

(b) \( e = \frac{|W|}{|Q_H|} = \frac{2200 \text{ J}}{6500 \text{ J}} = 0.34 = 34 \%\)

20.7

For an Otto cycle with \( \gamma = 1.40 \) and \( r = 9.50 \), the temperature of the gasoline-air mixture when it enters the cylinder is 22.0 °C (point a of Fig. 20.5). (a) What is the temperature at the end of the compression stroke (point b)? (b) The initial pressure of the gasoline-air mixture (point a) is 8.40 \( \times 10^4 \) Pa, slightly below atmospheric pressure. What is the pressure at the end of the compression stroke?

(a) The compression stroke from point a to point b is an adiabatic compression. The initial absolute temperature at point a is \( T_i = (22.0 + 273.2) \text{ K} = 295.2 \text{ K} \). The final temperature \( T_f \) at point b is

\[ T_f = T_i \left( \frac{V_i}{V_f} \right)^{\gamma - 1} = T_i \left( \frac{V_i}{V_f} \right)^{1.40 - 1} = (295 \text{ K})(9.50)^{1.40 - 1} = 726 \text{ K} . \]

(b) \( p_f = p_i \left( \frac{V_i}{V_f} \right)^\gamma = p_i r^\gamma = (8.40 \times 10^4 \text{ Pa})(9.50)^{1.40} = 1.96 \times 10^6 \text{ Pa} . \)
A refrigerator has a coefficient of performance of 2.10. Each cycle it absorbs $3.40 \times 10^4$ J of heat from the cold reservoir. (a) How much mechanical energy is required each cycle to operate the refrigerator? (b) During each cycle, how much heat is discarded to the high-temperature reservoir?

(a) $|W| = \frac{|Q_{cl}|}{K} = \frac{3.40 \times 10^4 \text{ J}}{2.10} = 1.62 \times 10^4 \text{ J}.$

(b) $K = \frac{|Q_{cl}|}{|W|} = \frac{|Q_{cl}|}{|Q_{Hl} - |Q_{cl}||}$, so $\frac{1}{K} = \frac{|Q_{Hl} - |Q_{cl}||}{|Q_{cl}|} = 1$ and $|Q_{Hl}| = |Q_{cl}| \left( \frac{1}{K} + 1 \right) = (3.40 \times 10^4 \text{ J}) \left( \frac{1}{2.10} + 1 \right) = 5.02 \times 10^4 \text{ J}.$

A Carnot engine whose high-temperature reservoir is at 620 K takes in 550 J of heat at this temperature in each cycle and gives up 335 J to the low-temperature reservoir. (a) How much mechanical work does the engine perform during each cycle? (b) What is the temperature of the low-temperature reservoir? (c) What is the thermal efficiency of the cycle?

(a) $|W| = |Q_{Hl} - |Q_{cl}|| = 550 \text{ J} - 335 \text{ J} = 215 \text{ J}.$

(b) $T_C = T_H \frac{|Q_{cl}|}{|Q_{Hl}|} = \left( \frac{335 \text{ J}}{550 \text{ J}} \right) 620 \text{ K} = 378 \text{ K}.$

(c) $e = 1 - \frac{T_C}{T_H} = 1 - \frac{378 \text{ K}}{620 \text{ K}} = 0.390 = 39.0 \%.$

(a) Show that the efficiency $e$ of a Carnot engine and the coefficient of performance $K$ of a Carnot refrigerator are related by $K = (1 - e)/e$. The engine and refrigerator operate between the same hot and cold reservoirs. (b) What is $K$ for the limiting values $e = 1$ and $e = 0$? Explain.

(a) $K = \frac{T_C}{T_H - T_C} = \frac{T_C/T_H}{1 - (T_C/T_H)}$ \hspace{1cm} (1)

$e = 1 - \frac{T_C}{T_H}$, so $\frac{T_C}{T_H} = 1 - e$ \hspace{1cm} (2)

Substituting (2) into (1) gives $K = \frac{1 - e}{1 - (1 - e)} = \frac{1 - e}{e}.$

(b) An engine efficiency $e = 1$ gives $K = 0$. This is good for a heat engine (but impossible according to the engine statement of the second law of thermodynamics) because that would mean that all of the heat input from the high-temperature reservoir is converted into work. But that is not good for a refrigerator because all the work input gets converted into heat which goes into the high-temperature reservoir and none of it is used to transport heat from the low-temperature reservoir inside the refrigerator to the high-temperature reservoir outside the refrigerator, and thus there is no refrigeration.

On the other hand, a zero efficiency engine $e = 0$ gives $K = \infty$. A zero efficiency engine means that none of the heat input from the high-temperature reservoir is converted into work, which completely defeats the purpose of heat engines, and all of it is dumped into the low-temperature reservoir. But this would be good for refrigerators because it would mean that zero work would be required to transport heat from the low-temperature reservoir inside the refrigerator to the outside; this is a flagrant violation of the refrigerator statement of the second law of thermodynamics which states that heat cannot flow from a low-temperature system to a higher-temperature system without the input of work. Heat spontaneously flows instead from hot to cold objects when they are brought into thermal contact.
You pour 100 g of water at 80.0 °C into the ocean, which is at 20.0 °C, and wait about ten minutes. Treat the water you pour and the ocean as an isolated system. (a) Is this process reversible or irreversible? Explain your reasoning using simple physical reasoning without resorting to any equations. (b) Calculate the net entropy change of the system during this process. Explain whether or not this result is consistent with your answer to part (a).

(a) The process is clearly irreversible, because there is no way to reverse the process by any small change in the process parameters. In this case, there is no way to reverse the pouring operation and unmix the poured water from the ocean.

(b) For the poured water of mass $m_{\text{water}} = 0.100 \text{ kg}$, the initial temperature is $T_i = (80.0 + 273.2) \text{ K} = 353.2 \text{ K}$ and the final temperature is the temperature of the ocean $T_f = (20.0 + 273.2) \text{ K} = 293.2 \text{ K}$. The ocean temperature does not change measurably after the water is poured into it because it has such a huge heat capacity compared to that of the poured water. The entropy change of the poured water as it is cooled to the ocean temperature is

$$\Delta S_{\text{water}} = \int_{T_i}^{T_f} \frac{dQ}{T} = m_{\text{water}} c_T \int_{T_i}^{T_f} \frac{dT}{T} = m_{\text{water}} c_T \ln\left(\frac{T_f}{T_i}\right) = (0.100 \text{ kg})(4190 \text{ J/kg}) \ln\left(\frac{293.2 \text{ K}}{353.2 \text{ K}}\right) = -78.0 \text{ J/K}.$$

The heat absorbed by the water is

$$Q_{\text{water}} = m_{\text{water}} c_T (T_f - T_i) = (0.100 \text{ kg})(4190 \text{ J/kg})(293.2 \text{ K} - 353.2 \text{ K}) = -2.51 \times 10^4 \text{ J}.$$

We consider the poured water and the ocean to be an isolated system, so the heat absorbed by the water and the heat absorbed by the ocean add to zero. The change in entropy of the ocean is

$$\Delta S_{\text{ocean}} = -\frac{Q_{\text{water}}}{T_{\text{ocean}}} = \frac{-2.51 \times 10^4 \text{ J}}{293 \text{ K}} = 85.7 \text{ J/K}.$$

The net change in entropy of the system is

$$\Delta S = \Delta S_{\text{water}} + \Delta S_{\text{ocean}} = -78.0 \text{ J/K} + 85.7 \text{ J/K} = +7.7 \text{ J/K}.$$

Thus the net entropy of the system increases due to the water poured into the ocean, which is the earmark of an irreversible thermodynamic process, consistent with the answer to part (a).

Two moles of an ideal gas undergo a reversible isothermal expansion from 0.0280 m$^3$ to 0.0420 m$^3$ at a temperature of 25.0 °C. What is the change in entropy of the gas?

In general the entropy change of an ideal gas for any change of temperature and/or volume is

$$\Delta S = n C_V \ln\left(\frac{T_f}{T_i}\right) + n R \ln\left(\frac{V_f}{V_i}\right).$$

In the present case, the first term on the right hand side is zero because the temperature is constant. Thus,

$$\Delta S = n R \ln\left(\frac{V_f}{V_i}\right) = (2.00 \text{ mol})(8.32 \text{ J/mol·K}) \ln\left(\frac{0.0420 \text{ m}^3}{0.0280 \text{ m}^3}\right) = 6.75 \text{ J/K}.$$